William F. Le Fevre,[1] *M.S., Professional in C.E.*

Speed Analysis of Passenger Cars in Free-Fall Launch Motions

REFERENCE: Le Fevre, W. F., "**Speed Analysis of Passenger Cars in Free-Fall Launch Motions,**" *Journal of Forensic Sciences*, JFSCA, Vol. 31, No. 3, July 1986, pp. 886-902.

ABSTRACT: Applied forensic engineering in the field of accident reconstruction is often required to determine vehicle speeds in crash and collision cases. One type of automobile crash is that in which a car becomes airborne after being launched from an abrupt change of ground contour. This treatise covers an analysis of speed at launch based on measured distances from a launch surface to the landing point and on the slope of the launch surface. A refinement is introduced accounting for the effect of pitch motion of the vehicle to obtain a more accurate evaluation of speed.

KEYWORDS: engineering, accidents, automobiles, collision research

Terminology

- a Linear acceleration
- B Wheelbase
- b Distance from mass center to rear wheels
- c Distance from mass center to front wheels
- d Distance from front wheels
- f Distance from mass center to landing contact point on vehicle
- F Linear force
- g Acceleration of gravity
- h Distance ground to underside of vehicle
- h_c Distance ground to mass center
- I Moment of inertia, pitch mode
- M Mass
- R Reaction force
- S Linear displacement
- t Time
- T Torsional moment
- V Velocity
- W Weight
- x x-direction, Cartesian coordinate
- y y-direction, Cartesian coordinate
- α Angular acceleration, pitch mode
- ω Angular velocity, pitch mode
- θ Angular displacement, pitch mode
- ϕ Angle of tangent to parabolic trajectory of mass center from level plane

Received for publication 31 Aug. 1985; accepted for publication 13 Nov. 1985.
[1]Consulting engineer, William F. Le Fevre Co., Cleveland, OH.

Evaluation of vehicle speed is often necessary in reconstructing sequential events in crash and collision cases. One type of crash is that in which a car becomes airborne after being launched from an abrupt change in ground contour. Baker [1] has presented a method of calculating the vehicle speed at launch on the basis that the mass center of the car follows a trajectory of a projectile. Such an analysis of speed is based on the horizontal distance the car traveled, the vertical distance it fell, and on the slope of the surface from which it was launched, and that the falling motion begins when the mass center of the vehicle is over the edge of the launch surface. Apparently, there is an assumption in Baker's solution that the car maintains an attitude parallel to the slope of the launch platform throughout its airborne motion.

It is a matter of observation of the motion of cars purposely jumped in stunt shows and of free-falling cars in movie and video shows that the car develops a pitch motion on launch such that the front end of the car appears to fall faster than the rear. Since speed calculations are based on measurements to impact points of the car upon landing, the rotation of the car in pitch motion can influence the indicated landing point and, consequently, the accuracy of the speed calculations.

As a singular example, assume a vehicle strikes an embankment head-on after launch as shown in Fig. 1. If it is assumed that the vehicle remains horizontal after launch from a horizontal platform, both the horizontal distance and, more importantly, the vertical distance of fall of the mass center will be influenced if by the time of impact the car has developed a significant pitch displacement.

While the Baker method is simple to apply, it is worthwhile to investigate the effect of the pitch motion on the analysis of launch speed. A presentation is made in the following leading to the solution of the car speed at launch beginning at the instant the front wheels lose ground support to the landing of the car, whether against an embankment or on some other surface, and which accounts for the effect of pitch motion on the displacement of the mass center.

Launch Motions

Position 0 to Position 1—Start of Launch

Consider a car with a weight W and a pitch moment of inertia I about a mass center located a distance b from the rear wheels and at a height h_c above the ground and a wheelbase B as diagramed in Fig. 2.

The front wheels are at the edge of a launch surface that has a slope m which is positive if upward in the direction of the vehicle motion at an initial velocity V. The conditions of motion are:

Vertical

$$\text{Acceleration,} \quad a_{y0} = 0 \tag{1}$$

$$\text{Velocity,} \quad V_{y0} = mV \tag{2}$$

Horizontal

$$\text{Acceleration,} \quad a_{x0} = 0 \tag{3}$$

$$\text{Velocity,} \quad V_{x0} = V \tag{4}$$

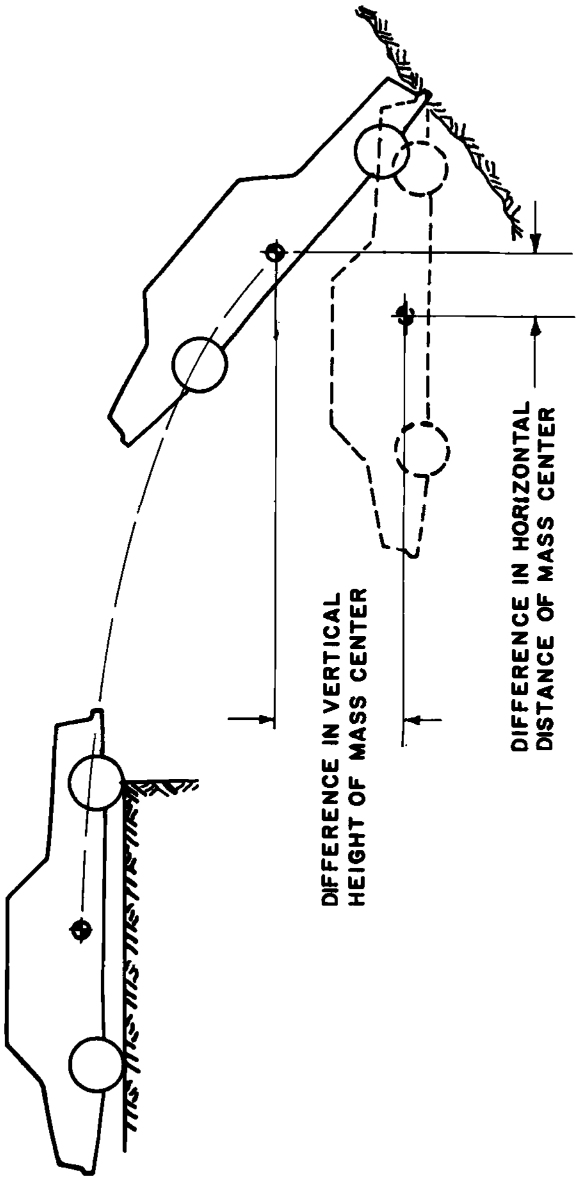

FIG. 1—*Vehicle strikes an embankment head-on after launch.*

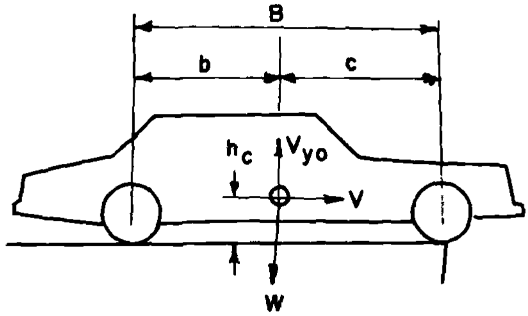

FIG. 2—*Front wheels of vehicle are at the edge of a launch surface.*

Pitch

$$\text{Angular acceleration,} \quad \alpha_0 = 0 \tag{5}$$

$$\text{Angular velocity,} \quad \omega_0 = 0 \tag{6}$$

Position 1—Rear Wheels at Edge of Launch Surface

Assume that the car has moved to a position at which the rear wheels are about to exit the ramp as diagramed in Fig. 3.

The suspension and tires at the rear wheels constitute spring elements in series, so the load on the rear wheels remains practically unchanged after the front wheels lose support upon exiting the ramp (see Appendix A). In view of the short time the load remains on the rear wheels during an actual launch, it is assumed that during the launch period, the ground reaction at the rear wheels is unchanged. The conditions of motion between Positions 0 and 1 are:

Vertical

Summing forces in the y-direction;

$$\Sigma F_y = Ma_y; \qquad \Sigma F_y = W - W_r = W - \frac{Wc}{B} = W\left(1 - \frac{c}{B}\right)$$

$$W\left(1 - \frac{c}{B}\right) = \frac{W}{g}a_y$$

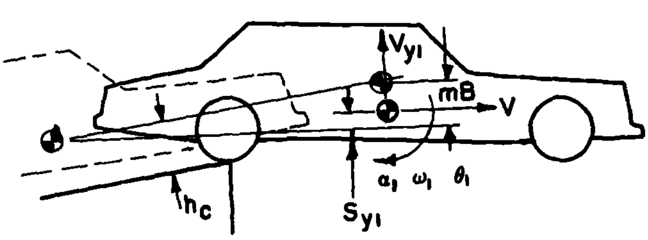

FIG. 3—*Position 1—start of free-fall motion.*

Then,

$$\text{Acceleration,}\quad a_{y1} = g(1 - c/B) \tag{7}$$

$$\text{Velocity,}\quad V_{y1} = V_{y0} + a_{y1}t_1$$

$$t_1 = B/V$$

$$= mV - gB(1 - c/B)/V$$

$$= mV - g(B - c)/V \tag{8}$$

$$\text{Displacement,}\quad S_{y1} = V_{y0}t_1 + \frac{a_{y1}t_1^2}{2}$$

$$= \frac{mVB}{V} - \frac{gB^2}{2V^2}(1 - c/B)$$

$$= mB - \frac{gB^2}{2V^2}\left(\frac{B - c}{B}\right)$$

$$= B\left[m - \frac{g(B - c)}{2V^2}\right] \tag{9}$$

Horizontal

$$\text{Acceleration,}\quad a_{x1} = 0 \tag{10}$$

$$\text{Velocity,}\quad V_{x1} = V \tag{11}$$

$$\text{Displacement,}\quad S_{x1} = B \tag{12}$$

Pitch

Summing moments about rear wheels;

$$\Sigma T = Wb = I\alpha + Ma_y b$$

$$= I\alpha + \frac{Wa_y b}{g}$$

Angular acceleration

$$\alpha = \frac{Wb - \dfrac{Wa_y b}{g}}{I}$$

$$= \frac{Wb(1 - a_y/g)}{I}$$

$$= \frac{Wbc}{BI} \tag{13}$$

Assume a sharp-edged end of a launching platform. At a velocity of V, the time the rear wheels exit the ramp after the front wheels is

$$t_1 = B/V \tag{14}$$

Thus, by the time the rear wheels become airborne, the car will have a pitch motion velocity of

$$\omega_1 = \alpha t_1 = \frac{Wbc}{IV} \tag{15}$$

At the end of the launch period when the rear wheels leave the platform, the pitch displacement of the car is

$$\theta_1 = \frac{\alpha t_1^2}{2} = \frac{BWbc}{2IV^2} \tag{16}$$

and the pitch angle from the horizontal is

$$\theta_1' = m - \theta_1 \tag{17}$$

Position 1 to Position 2—Airborne Motions

Consider the airborne motion of a car from the time the rear wheels leave the launch platform until the car lands at a horizontal distance D and at a vertical distance H measured from the edge of the launch ramp as diagramed in Fig. 4.

The conditions of motion are:

Vertical

Acceleration, $$a_y = g \tag{18}$$

Velocity,
$$\begin{aligned} V_{y2} &= V_{y1} + a_y t_{1-2} \\ &= mV - \frac{gB}{V}(1 - c/B) - gt_{1-2} \\ &= mV - \frac{g}{V}(B - c) - gt_{1-2} \end{aligned} \tag{19}$$

Displacement,
$$\begin{aligned} S_{y1-2} &= V_{y1}t_{1-2} + \frac{a_y}{2}(t_{1-2})^2 \\ &= t_{1-2}\left[mV - \frac{g}{V}(B - c)\right] - \frac{g}{2}(t_{1-2})^2 \end{aligned} \tag{20}$$

Horizontal

Acceleration, $$a_{x2} = 0 \tag{21}$$

Velocity, $$V_{x2} = V \tag{22}$$

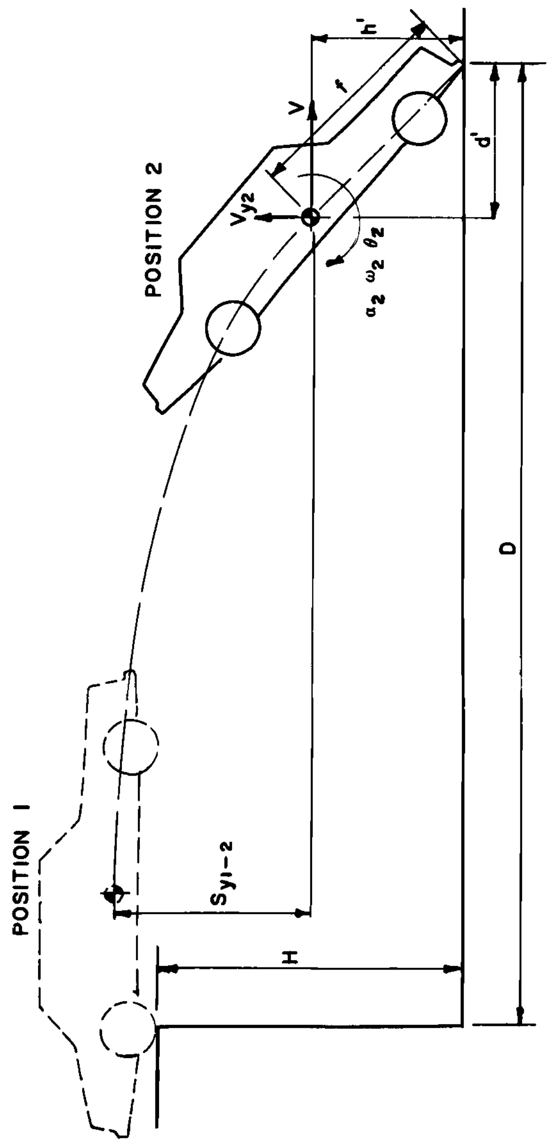

FIG. 4—*Position 2—landing from free fall.*

$$\text{Displacement,} \qquad S_{x1-2} = Vt_{1-2} \tag{23}$$

Pitch

$$\text{Angular acceleration,} \qquad \alpha = 0 \tag{24}$$

$$\text{Angular velocity,} \qquad \omega = \frac{Wbc}{IV} \tag{25}$$

$$\text{Angular displacement,} \qquad \theta_2 = \theta_1 + \omega t_{1-2}$$

$$= \frac{WBbc}{2IV^2} + \frac{Wbc}{IV} t_{1-2}$$

$$= \frac{Wbc}{IV}\left(\frac{B}{2V} + t_{1-2}\right) \tag{26}$$

Angular displacement from horizontal

$$\theta_2' = m - \theta_2 \tag{27}$$

The elevation of the mass center above the ramp at Position 1 is $h_c + S_{y1}$. Now, summing the distances in the vertical direction,

$$H + h_c + S_{y1} = S_{y1-2} + h'$$

$$0 = S_{y1-2} + h' - S_{y1} - H - h_c$$

$$= t_{1-2}\left[mV - \frac{g}{V}(B - c)\right] - \frac{g}{2}(t_{1-2})^2 + f\theta_2' - B\left[m - \frac{g(B - c)}{2V^2}\right]$$

$$- H - h_c$$

$$0 = t_{1-2}\left[mV - \frac{g}{V}(B - c)\right] - \frac{g}{2}(t_{1-2})^2$$

$$+ f\left[m - \frac{Wbc}{IV}\left(\frac{B}{2V} + t_{1-2}\right)\right] - B\left[m - \frac{g(B - c)}{2V^2}\right] - H - h_c$$

$$0 = mVt_{1-2} - \frac{g}{V}(B - c)t_{1-2} - \frac{g}{2}(t_{1-2})^2 + mf - \frac{fWBbc}{2IV^2}$$

$$- Bm + \frac{Bg(B - c)}{2V^2} - H - h_c - \frac{fWbct_{1-2}}{IV}$$

$$0 = -\frac{g}{2}(t_{1-2})^2 + t_{1-2}\left[mV - \frac{g(B - c)}{V} - \frac{fWbc}{IV}\right] + mf$$

$$- \frac{fWBbc}{2IV^2} - Bm + \frac{Bg(B - c)}{2V^2} - H - h_c$$

Changing signs and solving for t_{1-2} by the quadratic equation,

$$t_{1-2} = \frac{1}{g}\left\{\left[mV - \frac{g(B-c)}{V} - \frac{fWbc}{IV}\right] + \sqrt{\left[mV - \frac{g(B-c)}{V} - \frac{fWbc}{IV}\right]^2 - 2g\left(m(B-f) + \frac{B}{2V^2}\left[\frac{fWbc}{I} - g(B-c)\right] + H + h_c\right)}\right\} \tag{28}$$

The sum of the distances in the horizontal direction is

$$\begin{aligned} D + c &= B + Vt_{1-2} + d' \\ &= B + Vt_{1-2} + f\cos(57.3\,\theta_2') \end{aligned} \tag{29}$$

and the pitch angle at landing is

$$\theta_2' = m - \theta_2 = m - \frac{Wbc}{IV}\left(\frac{B}{2V} + t_{1-2}\right) \tag{30}$$

Minimum Free-Fall Launch Velocity

Assume that the forward velocity V of the car has some value so the pitch motion of the car after the front wheels leave a sharp-edged platform causes an osculation between the underside of the car and the edge of the launch surface at some distance d from the front wheels without modifying the linear and pitch velocities. The pitch velocity of the car will be $\omega = \alpha t$ in which $t = d/V$. By Eq 13 the acceleration of pitch motion is

$$\alpha = \frac{Wbc}{BI}$$

so the angular velocity of pitch motion is

$$\omega = \frac{Wbcd}{BIV} \tag{31}$$

The pitch displacement at time t is

$$\theta = \frac{\omega t}{2} = \frac{Wbcd^2}{2BIV^2} \tag{32}$$

Letting h be the height of the underside of the car above ground level

$$h = \theta(B - d) = \frac{Wbc(B-d)d^2}{2BIV^2}$$

from which

$$V = \sqrt{\frac{Wbcd^2(B-d)}{2BIh}} \tag{33}$$

For any given set of vehicle properties, Eq 33 can be written in the form

$$V^2 = Kd^2(B - d) \tag{34}$$

which describes a curve shown in Fig. 5. Rewriting Eq 34 in the form

$$Y = (KBX^2 - KX^3)^{1/2}$$

and differentiating

$$\frac{d}{dX}(KBX^2 - KX^3)^{1/2} = \frac{2KBX - 3KX^2}{2(KBX^2 - KX^3)^{1/2}}$$

and equating to zero

$$2BX = 3X^2$$

$$X = 2B/3 \tag{35}$$

which means the point of osculation is two thirds the wheelbase for any car configuration for the condition of minimum velocity to produce a free-fall launch.

Substituting Eq 35 in Eq 33, the minimum velocity to produce a free-fall launch is

$$V = \sqrt{\frac{4WB^2bc}{54Ih}} \tag{36}$$

Problem Solving

A solution for launch velocity considered in the foregoing depends upon the field measurements D, H, and m. After these values are made available, the time t_{1-2} can be calculated by Eq 28 using an assumed value of velocity, known H and m. This value of t_{1-2} can be substituted in Eq 30 to calculate the pitch angle at landing, θ_2'. The pitch angle can then be substituted in Eq 29 to calculate a value for D. Repeating for other values of V, a plot of V

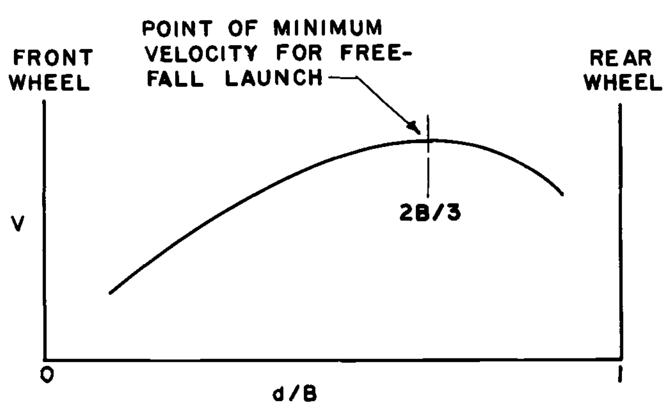

FIG. 5—*Curve for minimum velocity to produce a free-fall launch.*

versus D can be made for various values of assumed V. The intersection with the measured value of D establishes the launch velocity V.

Pitch Displacement in Relationship to Launch Velocity

It is of some interest, at least academically, to examine the relationship of pitch displacement in the free-fall condition as a function of launch velocity. One means of comparison lies in the relationship of pitch displacement to the tangent of the free-fall trajectory curve at a given instant. Consideration is given to the parabolic trajectory of the mass center in the airborne motion in Appendix B and to the evaluation of the slope of the tangent. Assuming an automobile having the properties listed in Table 1, the slope of the trajectory curve can be calculated by Eq B-9 and compared to the pitch displacement obtained by Eq 27.

The differences between the pitch displacement angle and tangent angle for the case of minimum launch speed by Eq 36 is shown for various values of linear displacement in Table 2 and for the particular case of a level launch platform ($m = 0$). Similar values for velocities of 44 and 88 ft/s (13 and 27 m/s) are given in Tables 3 and 4.

The comparisons given in Tables 2, 3, and 4 show that at low speeds, the pitch displacement increases faster than the curve tangent slope, but as the speeds increase, the differences lessen until at high speeds the pitch displacements and tangent angles are nearly the same even for large translation displacement. This is as it should be simply because the pitch velocity at Position 1 is a product of the angular acceleration expressed by Eq 13 and the dwell time of the rear wheels on the launch ramp, $t = B/V$.

TABLE 1—*Passenger car properties used in sample calculations.*[a]

Wheelbase	$B = 10$ ft
Front wheels to mass center	$c = 5$ ft
Rear wheels to mass center	$b = 5$ ft
Height of mass center	$h_c = 2$ ft
Underside ground clearance	$h = 0.8$ ft
Weight	$W = 3500$ lbs
Pitch moment of inertia	$I = 2717$ ft · lbf/s^2
Mass center to front end	$f = 8$ ft

[a]1 ft = 0.3048 m and 1 lb = 0.4536 kg.

TABLE 2—*Comparison of parabola tangent angle ϕ with pitch displacement angle θ, launch velocity 17.3 ft/s (5.3 m/s), launch slope* m = *0.*

Distance from Ramp, ft[a]	ϕ deg	θ deg	$\theta - \phi$ deg
5	28.3	30.8	2.5
6.73	35.9	41.5	5.6
8.46	42.3	52.1	9.8
10.19	47.6	62.8	15.2
11.92	52.0	73.5	21.5
13.63	55.7	84.1	28.4
15.38	58.9	94.8	35.9

[a]1 ft = 0.3048 m.

TABLE 3—*Comparison of parabola tangent angle ϕ with pitch displacement angle θ, launch velocity 44 ft/s (13 m/s), launch slope* m = *0.*

Distance from Ramp, ft[a]	ϕ deg	θ deg	$\theta - \phi$ deg
5	4.75	4.75	0
9.4	8.88	8.95	0.07
13.8	12.92	13.15	0.23
18.2	16.84	17.30	0.46
22.6	20.60	21.50	0.90
27.0	24.18	25.70	1.52
31.4	27.57	29.92	2.35
35.8	30.77	34.10	3.33
40.2	33.76	38.30	4.54
44.6	35.56	42.50	6.94
49.0	39.18	46.70	7.52
53.4	41.60	50.89	9.29
57.8	43.87	55.09	11.22

[a]1 ft = 0.3048 m.

TABLE 4—*Comparison of parabola tangent angle ϕ with pitch displacement angle θ, launch velocity 88 ft/s (27 m/s), launch slope* m = *0.*

Distance from Ramp, ft[a]	ϕ deg	θ deg	$\theta - \phi$ deg
5	1.19	1.19	0
13.8	3.28	3.28	0
22.6	5.36	5.38	0.02
31.4	7.44	7.48	0.04
40.2	9.49	9.57	0.08
49.0	11.52	11.67	0.15
57.8	13.51	13.77	0.26
66.6	15.48	15.87	0.39
75.4	17.40	17.96	0.56
84.2	19.30	20.06	0.76
93.0	21.14	22.16	1.02
101.8	22.94	24.25	1.31

[a]1 ft = 0.3048 m.

Influence of Pitch Motion on Speed Evaluation

Considering the precedence of the solution for launch speed given in Ref *1*, the results obtained by that procedure can be compared with results obtained by the methods suggested herein. Given m, D, and H, the equation for launch speed in miles per hour is given in Ref *1* as

$$V = \frac{2.74D}{\sqrt{mD - H}} \tag{37}$$

in which H is negative if the landing point is lower than the edge of the launch ramp.

TABLE 5—*Comparison of speeds. Distance* D *calculated for given speed and for* H = *20 ft (6 m) then substituted in Eq 37 to obtain comparative speed.*

Given Speed, mph[a]	Slope m	Speed, mph by Eq 37	Percent Difference
60	0.1	56.8	5.4
60	0	57.2	4.8
60	−0.1	59	1.7
30	0.1	28.2	6.2
30	0	27.3	8.9
30	−0.1	26.1	12.9
20	0.1	17.2	13.8
20	0	16.6	17.0
20	−0.1	15.8	21.0

[a]1 mph = 1.609 kph.

The value of D can be calculated for given values of H and m for a vehicle of known properties. This value of D can be substituted in Eq 37 to obtain a comparative speed. Comparison of speeds for the same values of D, H, and m and for a car having the properties listed in Table 1 are given in Table 5.

It can be seen from the comparisons in Table 5 that if the pitch motion is neglected, large errors of speed evaluation based on measured distances and slope can be expected particularly in low speed ranges. Speed calculations for low speeds near the minimum given by Eq 36 and determined by Eq 37 become invalid.

Conclusion

In the study performed in the preceding, a means was developed to evaluate the pitch velocity and pitch displacement of a car launched into an airborne trajectory from a platform of given slope. It was found that including the pitch motion in an analysis of launch speed based on measured distances to the landing point on either horizontal or vertical surfaces gave slightly higher speed values than if the pitch displacement is neglected and the car is considered to be a point mass. Including the pitch displacement provides a refinement to give a more accurate evaluation of launch speeds in vehicle crash analysis problems.

APPENDIX A

In additional discussion of the effect of tire and suspension flexibility on the rear wheel loading when the front wheels exit the launch ramp, it is of interest to consider the cases of an uniform plate of length L and width $L/2$ and weight W suspended at the corners in three different ways as shown in the diagram in Fig. 6. A comparison can be made of the accelerations when the right-hand support is suddenly removed.

For each case, the moment of inertia of the plate about its own mass center is

$$I = \frac{W}{12g}\left(L^2 + \frac{L^2}{4}\right) = \frac{5WL^2}{48g}$$

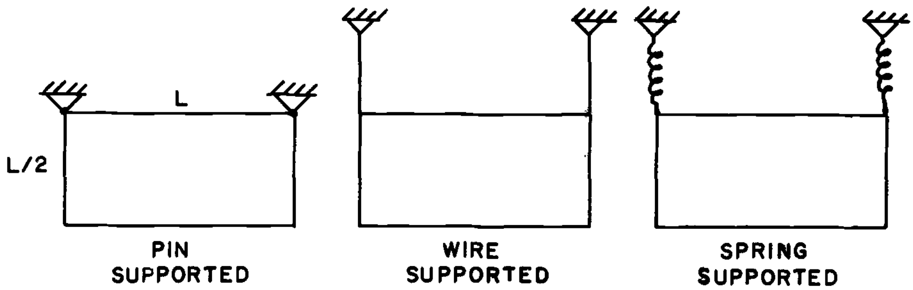

FIG. 6—*Uniform plate of length* L *and width* L/2 *and weight* W *suspended at the corners in three different ways.*

Pin-Supported Plate

Referring to the diagram in Fig. 7, the distance $r = \sqrt{5}L/4$, and as shown in the diagram,

$$\frac{WL}{2} = I\alpha + Mar$$

$$= \frac{5WL^2}{48g}\alpha + \frac{W}{g}\left(\frac{\sqrt{5}L\alpha}{4}\right)\left(\frac{\sqrt{5}L}{4}\right)$$

$$= \frac{5WL^2}{48g}\alpha + \frac{5WL^2}{16g}\alpha$$

$$\alpha = \frac{WL}{\dfrac{2WL^2}{g}\left(\dfrac{20}{48}\right)} = \frac{6g}{5L}$$

$$a = r\alpha = \frac{\sqrt{5}L}{4}\left(\frac{6g}{5L}\right) = \frac{3\sqrt{5}g}{10}$$

$$a_y = \left(\frac{3\sqrt{5}g}{10}\right)\frac{2}{\sqrt{5}} = .6g$$

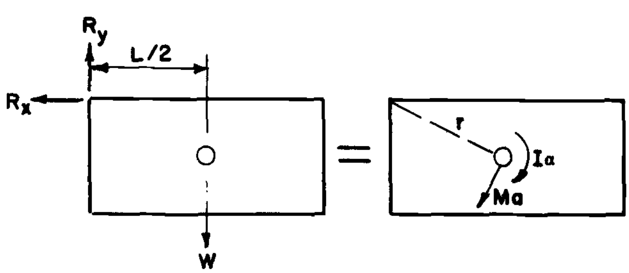

FIG. 7—*The kinetics of the pin-supported plate.*

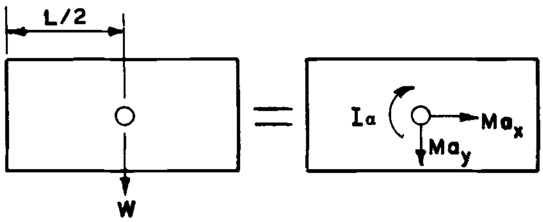

FIG. 8—*The kinetics of the wire-supported plate.*

Wire-Supported Plate

For the wire-supported plate, the kinetics can be diagramed as shown in Fig. 8 in which $\Sigma F_x = Ma_x = 0, a_x = 0$

$$\frac{WL}{2} = I\alpha + \frac{Ma_yL}{2} = \frac{5\,WL^2}{48g}\left(\frac{2a_y}{L}\right) + \frac{WLa_y}{2g}$$

$$= a_y\left(\frac{5\,WL}{24g} + \frac{WL}{2g}\right) = \frac{17\,WL}{24g}\,a_y$$

$$a_y = \frac{(WL/2)}{(17\,WL/24g)} = \frac{12}{17}g = .706g$$

Spring-Supported Plate

In the case of the spring-supported plate, the tension in the spring remains $W/2$ when the right-hand end is released because the elongation of the spring is unchanged. The kinetics can be diagramed as shown in Fig. 9.

$$\Sigma F_x = Ma_x = 0, \qquad a_x = 0$$

$$\Sigma F_y = Ma_y = \frac{Wa_y}{g} = W - W/2$$

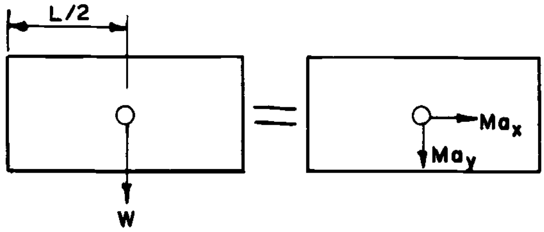

FIG. 9—*The kinetics of the spring-supported plate.*

$$\frac{W}{2} = \frac{W}{g}a_y$$

$$a_y = g/2 = .5g$$

These comparisons serve to justify the assumption that the load on the rear wheels remains unchanged after the front wheels leave the launch platform, particularly in view of the fact that the rear wheels exit the ramp in a comparatively short time after the front wheels.

APPENDIX B

Free-Flight Trajectory of a Vehicle Mass Center

Assuming that the drag forces of air resistance can be neglected, the free-flight trajectory of a vehicle mass center after launch can be defined by a parabolic curve. The general equation of a parabola is

$$X^2 = 4AY \tag{B-1}$$

for a parabola whose axis is parallel to the y-axis of Cartesian coordinates.

Referring to Fig. 10

$$S_x = Vt_0 \tag{B-2}$$

$$S_y = V_{y1} - gt_0^2/2 \tag{B-3}$$

in which t_0 is the time for the vertical velocity, V_{y1}, to become zero; that is,

$$t_0 = V_{y1}/g \tag{B-4}$$

Substituting t_0 in Eqs B-2 and B-3

$$S_x = \frac{VV_{y1}}{g} \tag{B-5}$$

$$S_y = \frac{V_{y1}^2}{g} - \frac{V_{y1}^2}{2g} = \frac{V_{y1}^2}{2g} \tag{B-6}$$

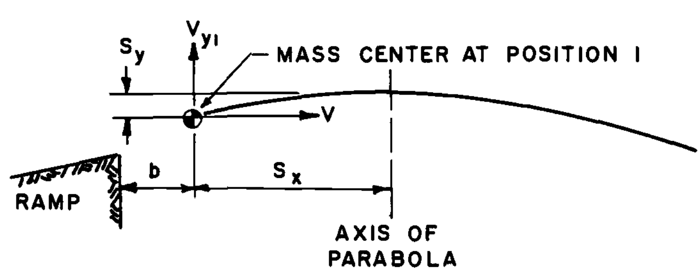

FIG. 10—*Free-flight trajectory of a vehicle mass center defined by a parabolic curve.*

Substituting Eqs B-5 and B-6 in Eq B-1

$$\left(\frac{VV_{y1}}{g}\right)^2 = 4A\left(\frac{V_{y1}^2}{2g}\right)$$

$$A = \frac{V^2}{2g} \tag{B-7}$$

so the equation for the free-fall trajectory is

$$X^2 = \frac{2V^2}{g}Y \tag{B-8}$$

Differentiating Eq B-8

$$\frac{dY}{dX} = \frac{2Xg}{2V^2} = \frac{Xg}{V^2} = \text{Tan } \phi \tag{B-9}$$

in which ϕ is the angle of the tangent at a given value of X on the parabola.

If the vertical velocity V_{y1} is positive at the start of the free-fall motion (Position 1), the value $S_x = VV_{y1}/g$ is positive and is measured in the direction of motion from Position 1. Correspondingly, if V_{y1} is negative, the axis of the parabola lies in a negative direction from Position 1. The portion of the parabola which lies ahead of Position 1 does not describe the path of the mass center between Positions 0 and 1.

Reference

[1] Baker, J. S., *Traffic Accident Investigation Manual*, The Traffic Institute, Northwestern University, Evanston, IL, 1975.

Address requests for reprints or additional information to
William F. Le Fevre
William F. Le Fevre Co.
Two Commerce Park Square
23200 Chagrin Blvd.
Cleveland, OH 44122